Question_1

[Solution]

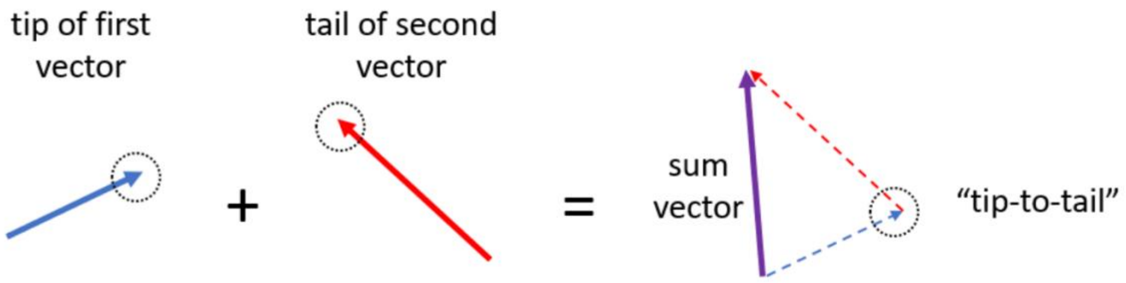
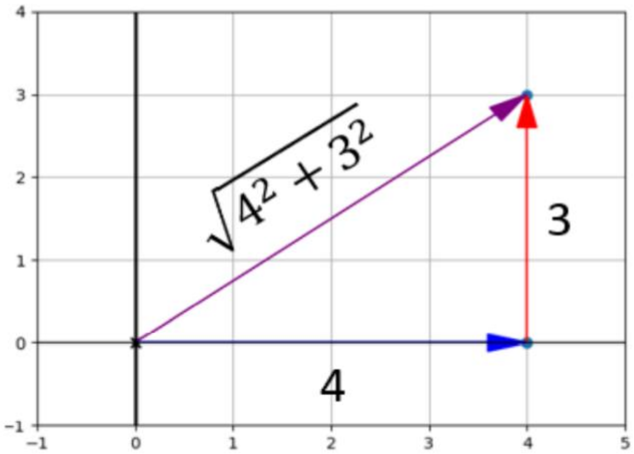


Figure 2.14 : "Tip-to-tail" addition of vectors.

Question_2

[Solution]

Using the Pythagorean theorem to find the length of a vector from the lengths of its *x* component and *y* component.



Question_3

[Solution]

With the vector u = (-2, 0), the vector v = (1.5, 1.5), and the vector w = (4, 1),

u + v = (-0.5, 1.5)

v + w = (5.5, 2.5)

u + w = (2, 1)

u + v + w = (3.5, 2.5)

Question_4

[Solution _Python]

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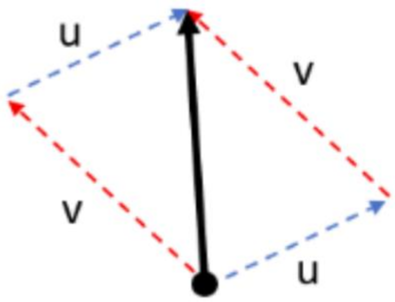
def add(*vectors):
    return (sum([v[0] for v in vectors]), sum([v[1] for v in vectors]))
```

Question_5

[Solution]

If you add two vectors u = (a,b) and v = (c,d), the coordinates a, b, c, and d are all real numbers. The result of the vector addition is u + v = (a+b, c+d). The result of v + u is (b+a, d+c), which is the same pair of coordinates since order doesn't matter when adding real numbers. Visually we can see this by adding an example pair of vectors tip-to-tail.



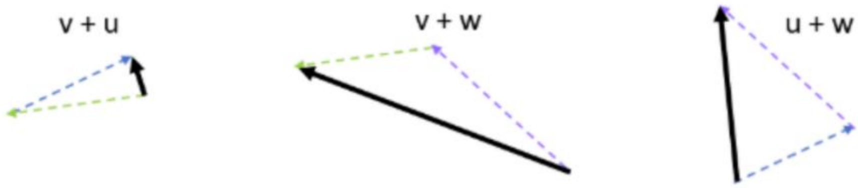
Tip-to-tail addition in either order yields the same sum vector.

It doesn't matter whether you add u + v or v + u (dashed), you get the same result vector (solid). In geometry terms, u and v defined a parallelogram and the vector sum is the length of the diagonal.

Question_6

[Solution]

We can measure each of the vector sums by placing the vectors tip-to-tail:

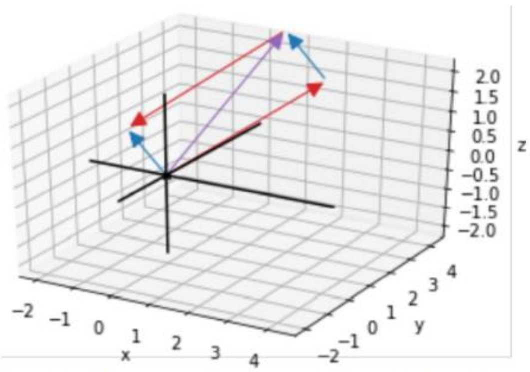


Inspecting the results, we can see that v+u is the shortest vector (u and v are in nearly opposite directions and come close to "cancelling each other out"). The longest vector is v+w.

Question_7

[Solution]

We can find the vector sum using the add function we built. Then to draw them tip-to-tail, we draw arrows from the origin to each point, and from each point to the vector sum (3,0,4). Like the 2D Arrow object, Arrow3D takes the "tip" vector of the arrow first, and then optionally the "tail" vector if it is not the origin.



Tip to tail addition shows (4,0,3) + (-1,0,1) = (-1,0,1) + (4,0,3) = (3,0,4).

Question_8

[Solution]

A(B(e1)) is A applied to B(e1) = (0,0,1) = e3. We already know A(e3) = (0,1,1) so B(A(e1)) = (0,1,1). A(B(e2)) is A applied to B(e2) = (2,1,0). This is a linear combination of A(e1), A(e2), and A(e3) with scalars (2,1,0): 2 · (1,1,1) + 1 · (1,0,-1) + 0 · (0,1,1) = (3,2,1). Finally, A(B(e3)) is A applied to B(e3) = (-1,0,-1). This is the linear combination -1·(1,1,1) + 0 · (1,0,-1) + -1 · (0,1,1) = (-1,-2,-2). Note that now we know the result of the composition of A and B for all of the standard basis vectors, so we can calculate A(B(v)) for any vector v.

Question_9

[Solution]

The dot product of the vector with the first row of the matrix is -2.5 · 1.3 + 0.3 · -0.7 = -3.46. The dot product of the vector with the second row of the matrix is -2.5 · 6.5 + 0.3 · 3.2 = -15.29. These are the coordinates of the output vector, so the result is:

$$\begin{pmatrix} 1.3 & -0.7 \\ 6.5 & 3.2 \end{pmatrix} \begin{pmatrix} -2.5 \\ 0.3 \end{pmatrix} = \begin{pmatrix} -3.46 \\ -15.29 \end{pmatrix}$$

Question_10

[Solution]

‘b’, This is a 3x5 matrix, since it has three rows and five columns.

Question_11

[Solution]

A) This product of a 2x2 matrix and a 4x4 matrix is not valid; the first matrix has two columns but the second matrix has four rows.

B) This product of a 2x4 matrix and a 4x2 matrix is valid; the four columns of the first matrix match the four rows of the second matrix. The result is a 2x2 matrix.

C) This product of a 3x1 matrix and a 1x8 matrix is valid; the single column of the first matrix matches the single row of the second. The result is a 3x8 matrix.

D) This product of a 3x3 matrix and a 2x3 matrix is not valid; the three columns of the first matrix do not match the two rows of the second.

Question_12

[Solution]

One possibility is to replace v = (-1,3) with a scalar multiple of itself, like (2, -6). The points of the form (2,2)+t·(- 1,3) agree with the points (2,2)+s·(2,-6) when t = -2·s. You can also replace u with any point on the line. Since (2,2) + 1·(-1,3) = (1,5) is on the line, (1,5) + t·(2,-6) is a valid equation for the same line as well.

Question_13

Solution. Let $B = \{\text{result is even}\} = \{2,4,6\}$ and $C = \{\text{result is a six}\} = \{6\}$. Then $\mathbb{P}(B) = \frac{1}{2}$ and $\mathbb{P}(C) = \frac{1}{6}$, but if I *know* that B has happened, then $\mathbb{P}(C|B)$ (read "the probability of C given B") is $\frac{1}{3}$ because given that B happened, we know the outcome was one of $\{2,4,6\}$ and since the die is fair, in the absence of any other information, we assume each of these is equally likely.

Now let $A = \{\text{result is divisible by } 3\} = \{3,6\}$. If we know that B happened, then the only way that A can also happen is if the outcome is in $A \cap B$, in this case if the outcome is $\{6\}$ and so $\mathbb{P}(A|B) = \frac{1}{3}$ again which is $\mathbb{P}(A \cap B)/\mathbb{P}(B)$.

Question_14

Solution. Write A_i for the event that the ith ticket drawn is from the set $\{C, A, L, V, I, N\}$. By (1.4),

$$\mathbb{P}(A_1 \cap \ldots \cap A_6) = \frac{6}{26} \cdot \frac{5}{25} \cdot \frac{4}{24} \cdot \frac{3}{23} \cdot \frac{2}{22} \cdot \frac{1}{21}.$$

Question_15

Solution. Using the definition of conditional probability,

$$\mathbb{P}(0 \text{ sent} \mid 0 \text{ received}) = \frac{\mathbb{P}(0 \text{ sent and } 0 \text{ received})}{\mathbb{P}(0 \text{ received})}.$$

Now

$$\mathbb{P}(0 \text{ received}) = \mathbb{P}(0 \text{ sent and } 0 \text{ received}) + \mathbb{P}(1 \text{ sent and } 0 \text{ received}).$$

Now we use (1.3) to get

$$\begin{aligned}\mathbb{P}(0 \text{ sent and } 0 \text{ received}) &= \mathbb{P}(0 \text{ received} \mid 0 \text{ sent})\mathbb{P}(0 \text{ sent}) \\ &= \big(1 - \mathbb{P}(1 \text{ received} \mid 0 \text{ sent})\big)\mathbb{P}(0 \text{ sent}) \\ &= \left(1 - \frac{1}{8}\right)\frac{4}{7} = \frac{1}{2}.\end{aligned}$$

Similarly,

$$\begin{aligned}\mathbb{P}(1 \text{ sent and } 0 \text{ received}) &= \mathbb{P}(0 \text{ received} \mid 1 \text{ sent})\mathbb{P}(1 \text{ sent}) \\ &= \frac{1}{6} \cdot \frac{3}{7} = \frac{1}{14}.\end{aligned}$$

Putting these together gives

$$\mathbb{P}(0 \text{ received}) = \frac{1}{2} + \frac{1}{14} = \frac{8}{14}$$

and

$$\mathbb{P}(0 \text{ sent} \mid 0 \text{ received}) = \frac{\frac{1}{2}}{\frac{8}{14}} = \frac{7}{8}.$$

Question_16

Solution. Write B for the event that the individual has the condition, and A for the event that the test outcome is positive. We are asked to calculate $\mathbb{P}(B|A)$.

We have $\mathbb{P}(B) = 1/1000$, $\mathbb{P}(A|B) = 1$ and $\mathbb{P}(A|B^c) = 0.01$.

Using the odds form of Bayes' Theorem given above, we could write

$$\begin{aligned} \frac{\mathbb{P}(B|A)}{\mathbb{P}(B^c|A)} &= \frac{\mathbb{P}(A|B)}{\mathbb{P}(A|B^c)} \frac{\mathbb{P}(B)}{\mathbb{P}(B^c)} \\ &= \frac{1}{0.01} \frac{1/1000}{999/1000} \\ &= 0.0999. \end{aligned}$$

Solving the equation $p/(1-p) = 0.0999$, we obtain $\mathbb{P}(B|A) \approx 0.091$. Even though the test is "99% accurate", nonetheless the conditional probability of having the condition given a positive test is still less than 10%. The chance of a a false positive considerably outweighs the chance of a true positive, since the prevalence in the population is very low.