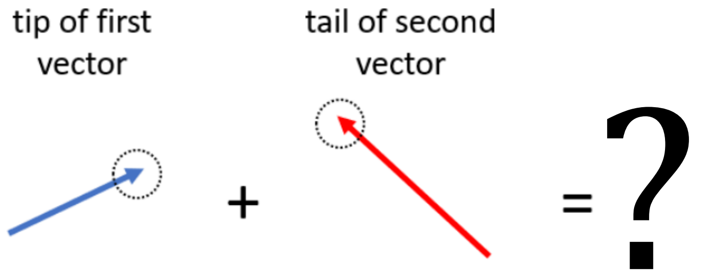
[Linear Algebra]

Question 1.

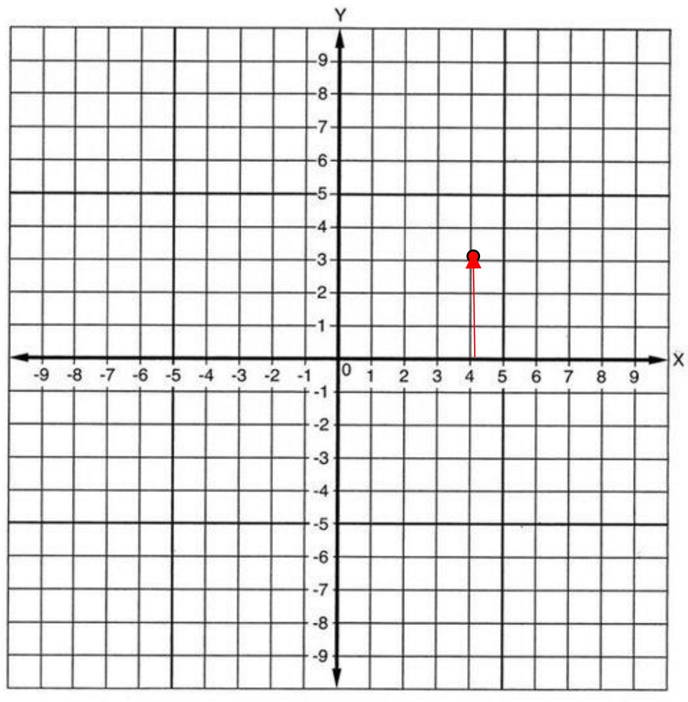
Please draw the sum vector from these two vectors below.



[Answer]

Question 2.

Please break the vector (4,3) into 2 vectors and compute the sum of these vectors.



[Answer]

Question 3.

If the vector u = (-2, 0), the vector v = (1.5, 1.5), and the vector w = (4, 1), what are the results of u + v, v + w, and u + w? What is the result of u + v + w?

[Answer]

Question 4. [Code question]

You can add any number of vectors together by summing all of their x coordinates and all of their y coordinates. For instance, the four-fold sum (1,2) + (2,4) + (3,6) + (4,8) has x component 1 + 2 + 3 + 4 = 10 and y component 2 + 4 + 6 + 8 = 20, making the result (10,20). Implement a revised add function that takes any number of vectors as arguments.

[Answer]

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Question 5.

Any sum of vectors v + w gives the same result as w + v. Explain why this is true using the definition of the vector sum on coordinates. Also, draw a picture to show why it is true geometrically.

[Answer]

Question 6.

Among the following three arrow vectors, labeled u, v, and w, which pair has the sum that gives the longest arrow? Which pair sums to give the shortest arrow?

[Answer]

Question 7.

Draw (4,0,3) and (-1,0,1) as Arrow3D objects, such that they are placed tip-to-tail in both orders in 3D. What is their vector sum?

[Answer]

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Question 8.

Suppose A and B are both linear transformations, with A(e1) = (1,1,1), A(e2) = (1,0,-1), and A(e3) = (0,1,1) and B(e1) = (0,0,1), B(e2) = (2,1,0), and B(e3) = (-1,0,-1). What is A(B(e_1)), A(B(e_2)), and A(B(e_3))?

[Answer]

Question 9.

What is the result of the following product of a 2-by-2 matrix with a 2D vector?

$$\begin{pmatrix} 1.3 & -0.7 \\ 6.5 & 3.2 \end{pmatrix} \begin{pmatrix} -2.5 \\ 0.3 \end{pmatrix}$$

[Answer]

Question 10.

What are the dimensions of this matrix?

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{pmatrix}$$

a.) 5x3

b.) 3x5

[Answer]

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Question 11.

Which of the following are valid matrix products? For those that are valid, what dimension is the product matrix?

A) $$\begin{pmatrix} 10 & 0 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 8 & 2 & 3 & 6 \\ 7 & 8 & 9 & 4 \\ 5 & 7 & 0 & 9 \\ 3 & 3 & 0 & 2 \end{pmatrix}$$

B) $$\begin{pmatrix} 0 & 2 & 1 & -2 \\ -2 & 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} -3 & -5 \\ 1 & -4 \\ -4 & -4 \\ -2 & -4 \end{pmatrix}$$

C) $$\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 3 & 3 & 5 & 1 & 3 & 0 & 5 & 1 \end{pmatrix}$$

D) $$\begin{pmatrix} 9 & 2 & 3 \\ 0 & 6 & 8 \\ 7 & 7 & 9 \end{pmatrix} \begin{pmatrix} 7 & 8 & 9 \\ 10 & 7 & 8 \end{pmatrix}$$

[Answer]

Question 12.

It turns out that the formula u + t·v is not unique; that is, you can pick different values of u and v and represent the same line. What is another line representing (2,2) + t·(-1,3)?

[Answer]

[Probability]

Question 13.

Suppose that in a single roll of a fair die we know that the outcome is an even number. What is the probability that it is in fact a six?

[Answer]

Question 14.

A bag contains 26 tickets, one with each letter of the alphabet. If six tickets are drawn at random from the bag (without replacement), what is the chance that they can be rearranged to spell CALVIN ?

[Answer]

Question 15.

A bitstream when transmitted has

$$\mathbb{P}(0 \ \ sent) = \frac{4}{7}, \qquad \mathbb{P}(1 \ \ sent) = \frac{3}{7}.$$

Owing to noise,

$$\mathbb{P}(1 \ \ received \mid 0 \ \ sent) = \frac{1}{8},$$
$$\mathbb{P}(0 \ \ received \mid 1 \ \ sent) = \frac{1}{6}.$$

What is P(0 sent | 0 received)?

[Answer]

Question 16.

A particular medical condition has prevalence 1/1000 in the population. There exists a test for the condition which has false negative rate 0 (i.e. any person with the condition will test positive) and false positive rate 0.0 (i.e. a typical person without the condition tests positive with probability 0.01). A member of the population selected at random takes a test, and tests positive. What is the probability that they have the condition?

[Answer]

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